# Problems that I would like Somebody to Solve

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## Contents

# 1 What is this?

This is a (very informal) collection of problems that are of personal interest to me, most of which are lesser known problems. Feel free to reach out to me if you have any questions regarding these questions and/or if you spot any glaring typos. I also recommend reaching out to me if you start seriously working on any of these problems so that I don't accidentally work on it at the same time (which has happened before!).

**Organization**. I have split these problems into three tiers based on my level of interest in these problems. The Triple diamond problems in Section 2 are the one's that keep me up at night, followed by the double diamonds in Section 3 and the single diamonds in Section 4. Some problem writeups have subproblems which might be of a different rank, which will be indicated in the problem statement. Section 5 contains the "Hall of Fame" list of solvers to past problems.

The problems appear is roughly reverse chronological order. Note that problems can jump between different tiers of interest depending on my mood, and I may remove problems from the list if I start actively working on them again.

**Acknowledgments**. We thank Zachary Chase and Zach Hunter for pointing out some small typos.

# 2 $\diamondsuit\diamondsuit\diamondsuit$ Problems

## 2.1 Small Quasikernels

Let $D$ be a digraph. Given a set $S$, we define $N^+(S) = \bigcup_{v\in S} N^+(v)$, where $N^+(v)$ is the out-neighborhood of $v$. We say that a set $K \subseteq V(D)$ is a *kernel* of $D$ if (1) $N^+(K) \cap K = \emptyset$ (that is, $K$ is an independent set of the underlying graph of $D$), and (2) $N^+(K) \cup S = V(D)$ (that is, every vertex is either in $K$ or can be reached by a vertex in $K$ in one step).

Not every digraph has a kernel (take any directed cycle of odd length), but it is not too hard to prove that every digraph has a *quasikernel*. This is a set $Q \subseteq V(D)$ such that (1) $N^+(Q) \cap Q = \emptyset$ and such that (2) $N^+(N^+(Q)) \cup N^+(Q) \cup Q = V(D)$. That is, it is an independent set such that every vertex can be reached from $Q$ in at most two steps.

Given that every digraph has a quasikernel, it is natural to ask how small of a quasikernel one can find. One quickly realizes that it can be quite large: any source of $D$ must belong to a quasikernel of $D$. Thus the most natural setting to consider is when $D$ has no sources, and in this case the following was conjectured by P.L. Erdős and Székely.

**Conjecture 2.1.** *Every digraph $D$ with no sources has a quasikernel of size at most $|V(D)|/2$.*

Overall very little is known here. There are a few special classes of digraphs for which this is known (see this survey [15] for more), and a very weak bound of $|V(D)| - \lfloor |V(D)|^{1/2} \rfloor$ is known in general [22], but outside of this we know basically nothing. One obstacle to this conjecture is the following problem.

**Conjecture 2.2.** *There exists some $\epsilon > 0$ such that every digraph $D$ contains a quasikernel $Q$ with*

$$|N^+[Q]| \geq \epsilon |V(D)|,$$

*where $N^+[Q] := Q \cup N^+(Q)$.*

It's shown in [22] that the conjecture above is a weaker conjecture than proving any non-trivial linear upper bound $(1-\epsilon)|V(D)|$ for the size of the smallest quasikernel in a source-free digraph. At the moment only the bound $|N^+[Q]| \geq |V(D)|^{1/3}$ is known, though this can likely be improved to $|V(D)|^{1/2}$. Many more open problems around this topic that I care about can be found in [22].

## 2.2 $C_4$-free Subgraphs of Random Hypergraphs

Given a hypergraph $H$ and a family of hypergrpahs $\mathcal{F}$, we define $\text{ex}(H, \mathcal{F})$ to be the maximum number of edges in an $\mathcal{F}$-free subgraph of $H$. We're particularly interested in the case when $H = G^r_{n,p}$, the random $r$-uniform hypergraph obtained by keeping each edge of $K^r_n$ independently and with probability $p$ and when $\mathcal{F}$ is a family of $r$-partite $r$-graphs.

Perhaps the simplest non-trivial case of this problem is when we consider $C_4$-free subgraphs of the random graph $G_{n,p}$. This problem was essentially solved by Füredi [11], and later two more solutions were given by Morris and Saxton [16] (who essentially solved the problem for both graph cycles and complete bipartite graphs). The problem in this section is concerned about extending these results to hypergraphs $C_4$'s; which remains an elusive problem despite having multiple proofs in the graph setting. There are many ways one can define what it means for a hypergraph to be a "$C_4$", below we consider two common notions.

Let $C^3_4$ be the 3-uniform loose 4-cycle, which can be defined by having edges

$$\{1,2,3\}, \{3,4,5\}, \{5,6,7\}, \{7,8,1\}.$$

That is, it's obtained from the graph $C_4$ by inserting an extra vertex into each edge. A standard deletion argument shows that, for example,

$$\mathbb{E}[\text{ex}(G^3_{n,n^{-2/3}}, C^3_4)] = \Omega(n^{4/3}),$$

and work of Nie [18] shows[1]

$$\mathbb{E}[\text{ex}(G^3_{n,n^{-2/3}}, C^3_4)] \leq n^{4/3+1/30+o(1)}.$$

**Problem 2.3.** *Improve either of these bounds for $\mathbb{E}[\text{ex}(G^3_{n,n^{-2/3}}, C^3_4)]$.*

Mubayi and Yepremyan [17] conjecture that the lower bound from the deletion argument is essentially best possible, which is known to be true if one looks at the analogous problem for uniformity at least 4 [17, 18].

[1] They proved bounds in a much larger range, but this is the point where the gap between the bounds is largest.

In another direction, we say that a 3-uniform hypergraph $F$ is a Berge $C_4$ if it has four edges $e_1, e_2, e_3, e_4$ and if there exist four distinct vertices $v_1, v_2, v_3, v_4$ with $v_i \in e_i \cap e_{i+1}$ for all $i$ (with the indices written cyclically). We let $\mathcal{B}_4^3$ denote the set of 3-uniform Berge $C_4$'s. It is known [18, 24] that

$$p^{1/4} n^{3/2-o(1)} \leq \mathbb{E}[\mathrm{ex}(G_{n,p}^3, \mathcal{B}_4^3)] \leq p^{1/5} n^{3/2+o(1)} \text{ for } p \gg n^{-2/3}.$$

**Problem 2.4.** *Improve either of these bounds for* $\mathbb{E}[\mathrm{ex}(G_{n,p}^3, \mathcal{B}_4^3)]$.

We note that improving the lower bound of Problem 2.3 is strictly easier than improving the lower bound of Problem 2.3, and if the lower bound of Problem 2.3 is tight (as conjectured by [17]), then it would be easier (in principle) to show that the lower bound for Problem 2.3 is tight.

# 3 ♢♢ Problems

## 3.1 Clique Supersaturation

Classic graph supersaturation problems ask: given an $n$-vertex graph with a given number of edges, how many copies of some other graph $F$ is $G$ guaranteed to have? Here we ask the same question but replace "number of edges" with "number of triangles". Our main question is the following.

**Conjecture 3.1.** *There exists $t_0$ such that for all $t \geq t_0$ and $1 \leq k \leq n^{1/2t}$, there exists an $n$-vertex graph $G$ with $\Omega(kn^{3/2})$ triangles and which has at most $k^t n^{3/2+o(1)}$ copies of $K_{2,t}$.*

It is known [7] that this bound would be best possible and that such graphs would exist if one could construct dense $C_8$-free subgraphs of $K_{m,n}$, but this is likely the wrong way to go about the problem. For $K_{3,t}$ we conjecture the following.

**Conjecture 3.2.** *For all $t \geq 3$ there exists a constant $k_0$ such that if $G$ is an $n$-vertex graph with $kn^2$ triangles and $k \geq k_0$ then $G$ contains at least $k^t n^{3-o(1)}$ copies of $K_{3,t}$.*

This bound would be best possible by considering $G_{n,p}$ for an appropriate value of $p$.

## 3.2 Squares of Eigenvalues

Given a graph $G$, let $\lambda_1, \ldots, \lambda_n$ denote the eigenvalue of its adjacency matrix and define

$$s^+(G) = \sum_{i:\lambda_i > 0} \lambda_i^2, \;\; s^-(G) = \sum_{i:\lambda_i < 0} \lambda_i^2.$$

The following curious conjecture was made by Elphick, Farber, Goldberg, and Wocjan [8]

**Conjecture 3.3.** *If $G$ is a connected $n$-vertex graph, then $s^+(G) \geq n-1$ and $s^-(G) \geq n-1$.*

Part of the motivation for this conjecture is that it's true (and tight) at the two "extremes" of connected graphs. Notably if $G$ is a tree then $s^+(G) = s^-(G) = n-1$, and if $G$ is a complete graph then $s^+(G) > s^-(G) = n-1$. The conjecture is notably known to be true for regular graphs [8], which excludes many plausible approaches for counterexamples.

One can also consider various varients of this conjecture where you replace $\lambda_i^2$ with $\lambda_i^r$, or by replacing the adjacency matrix $A$ with a "weighted" adjacency matrix. I have some notes on this and can share them upon request.

### 3.3 Eulerian Polynomials for Digraphs

Given a digraph $D$ and a bijection $\sigma : V(D) \to [n]$, we say that an arc $u \to v$ of $D$ is a descent of $\sigma$ if $\sigma(u) > \sigma(v)$. We let $\mathrm{des}(\sigma)$ denote the number of descents of $\sigma$ and define the generating function $A_D(t) = \sum_\sigma t^{\mathrm{des}(\sigma)}$.

Given a graph $G$, we define $\nu(G) = |A_D(-1)|$ where $D$ is any orientation of the edges of $G$ (and it turns out this is the same quantity regardless of how the edges are oriented).

**Problem 3.4.** *Give a combinatorial interpretation for $\nu(G)$ for all graphs $G$.*

For example, when $G$ is a path of order $n$, then $A_D(t)$ is (essentially) the well-studied Eulerian polynomial, and it is known here that $|A_D(-1)|$ is the number of alternating permutations of order $n$.

Somewhat more generally, in [4] we showed that for bipartite graphs, $\nu(G)$ is equal to the number of "even sequences" of $G$, i.e. the number of permutations $v_1, \ldots, v_n$ of its vertices so that the induced subgraphs $G[v_1, \ldots, v_i]$ all have an even number of edges. While there are examples of graphs where $\nu(G)$ is not equal to the number of even sequences (e.g. an odd cycle with a leaf), the number of even sequences always serves as an upper bound to $\nu(G)$, so perhaps $\nu(G)$ counts some subset of even sequences which have some special property.

Further questions related to $A_D(t)$ can be found in [4]. Here we present one more problem which isn't posted there. For any integer $n$, let $s_2(n)$ denote the number of 1's in the binary expansion of $n$. We showed that for any $n$-vertex digraph $D$, the multiplicity of $-1$ as a root in $A_D(t)$ is always at most $n - s_2(n)$ and we found a number of examples which show that this bound is tight. Curiously, we later found these same extremal examples showing up in a completely different context!

To this end, we say that a digraph $D$ is *impartial* is any two tournaments $T, T'$ on the same number of vertices has the same number of copies of $D$. A simple characterization of such digraphs was given by Zhao and Zhou [26], and in particular it is implicitly shown in [4] that every connected[2] impartial digraph achieves this $n - s_2(n)$ bound. We wonder if these are the only such examples.

**Conjecture 3.5.** *If $D$ is a connected $n$-vertex digraph such that $A_D(t)$ has $-1$ as a root with multiplicity $n - s_2(n)$, then $D$ is an impartial digraph.*

[2]Disjoint unions of impartial digraphs will also sometimes achieve this bound, but it's a little annoying to state this precisely.

To emphasize, there is no apriori reason to suspect that this is true just from the definition of things, and we would be very interested to know if there's some sort of deep connection going on here between these unrelated problems or if this is all just a mere coincidence.

## 3.4 Maximal Independent Sets of Clique-free Graphs

We say that a set of vertices $I \subseteq V(G)$ of a graph $G$ is a *maximal independent set*, or simply an MIS, if $I$ is an independent set but $I \cup \{v\}$ is not an independent set for any $v \notin I$. Let $m_t(n,k)$ denote the maximum number of MIS's of size $k$ that an $n$-vertex $K_t$-free graph can have.

We initiated the study of $m_t(n,k)$ together with He and Nie [12] (and we refer the reader to our paper for motivation of this particular problem). We stated a lot of open problems about this function in our paper, any of which I would be thrilled to see solved. Here we emphasize two of these problems.

Our first problem concerns upper bounding the number of MIS's in triangle-free graphs.

**Problem 3.6.** *Prove that there exists an integer $k \geq 5$ and real number $\epsilon > 0$ such that*

$$m_3(n,k) = O(n^{k-2-\epsilon}).$$

We conjectured that in fact $m_3(n,k) = \Theta(n^{\lfloor k/2 \rfloor})$ for all $k \geq 5$, and we implicitly proved $m_3(n,k) = O(n^{k-2})$ for $k \geq 5$. Thus this problem asks to improve our upper bound, which our conjectured lower bound suggests should be very far from tight as is.

The next problem concerns $K_4$-free graphs. In this setting we proved $m_4(n,3) \geq n^{2-o(1)}$ and that $m_4(n,3) = O(n^2)$.

**Problem 3.7.** *Determine whether the $o(1)$ term in the lower bound for $m_4(n,3)$ mentioned above is necessary or not.*

I believe that this $o(1)$ should be necessary. In fact, I believe that $m_4(n,3)$ should be equal (up to constants) to the maximum number of edges of an $n$-vertex graph which is such that every edge is contained in a unique triangle (determining this quantity is often referred to as the Ruzsa-Szemerédi problem). One approach that would give this stronger result is the following.

**Problem 3.8.** *Show that if $G$ is an $n$-vertex $K_4$-free graph with "many" (e.g. $n^{2-\epsilon}$) MIS's of size 3 such that every vertex is contained in at least one 3-MIS, then $\chi(G) = O(1)$.*

If this were true then one could essentially convert the problem of working with $K_4$-free graphs to working with tripartite graphs, and in this case we proved that the maximum number of 3-MIS's is essentially the solution to the Ruzsa-Szemerédi problem. We note that it's easy to prove that if $G$ is an $n$-vertex *triangle*-free graphs with at least one MIS of size $k$ that $\chi(G) \leq k+1$, and in particular for $K_4$-free graphs one may need much fewer than $n^{2-\epsilon}$ MIS's to guarantee a bounded chromatic number.

**Update**: I had previously asked Problem 3.8 without the assumption that every vertex be contained a 3-MIS, but a counterexample to this was found by Ramon I. Garcia. Specifically, one starts with an $n/2$ vertex tripartite graph $G_1$ with $n^{2-o(1)}$ 3-MIS's (e.g. by taking $G_1$ to be the complement of the Ruzsa-Szemerédi construction), then unions this with an $n/2$ vertex graph $G_2$ with high girth and chromatic number, then connects every vertex of the first part of $G_1$ with all of $G_2$.

# 4 ♢ Problems

## 4.1 Coloring mod $p$

Given a graph $G$ and an integer $p$, we say that $I \subseteq G$ is an *independent set mod* $p$ if every vertex in the induced graph $G[I]$ has degree $0 \mod p$. For example, independent sets are always independent sets mod $p$. We define the *mod* $p$ *independence number* $\alpha_p(G)$ to be the size of a largest independent set mod $p$. Similarly we define the *mod* $p$ *chromatic number* $\chi_p(G)$ to be the smallest integer $k$ such that there exists a partition $V_1 \cup \cdots \cup V_k$ of $V(G)$ such that $V_i$ is an independent set mod $p$ for all $i$.

**Conjecture 4.1.** *For all primes $p$, there exists a constant $C = C(p)$ such that for all graphs $G$, $\chi_p(G) \leq C$.*

It's quite plausible that the conjecture is true without having to restrict to primes, but focusing on primes is probably a good place to start since one can most easily use algebraic techniques in this case.

Gallai proved that Conjecture 4.1 is holds with $C = 2$ when $p = 2$, see [14] for a simple proof, as well as [10] for two other proofs written in a different language [3] Caro, Krasikov, and Roditty [3] proved a weaker version of Conjecture 4.1, showing that $G$ can be partitioned into $C$ induced subgraphs $G[V_1], \ldots, G[V_C]$ such that $e(G[V_i]) \equiv 0 \mod p$ for all $i$. Ferber, Hadiman and Krivelevich [9] showed that there exists a $C$ such that almost every graph has $\chi_p(G) \leq C$.

Overall Conjecture 4.1 seems pretty hard, and there are a couple of weaker versions of this conjecture that might be provable.

**Conjecture 4.2.** *For all primes $p$, there exists a constant $C = C(p)$ such that for all graphs $G$, $\alpha_p(G) \geq |V(G)|/C$.*

**Conjecture 4.3.** *For all primes $p$, there exists a constant $C = C(p)$ such that for all graphs $G$, one can partition $V(G)$ into $C$ sets $V_1 \cup \cdots \cup V_C$ such that no $G[V_i]$ contains a vertex of degree $1 \mod p$.*

It also natural to conjecture this for $-1 \mod p$, since in both cases we know the result holds for $p = 2$.

Lastly, we note that a trivial lower bound on the $C(p)$ in Conjecture 4.1 is $C(p) \geq p$ by considering $G = K_p$. However, for odd $p$ one can prove that we must have $C(p) \geq p+1$ (there are many examples; the simplest is to take a circulant graph on $2p+2$ vertices such that every

[3] This reference gives three proofs that there exists a solution to the "Lights Out!" game. It is relatively easy to show that this implies the stated result by considering a graph $G'$ with a leaf attached to each vertex.

vertex has degree $p+1$). It would be interesting to know if one could find constructions which give significantly stronger bonds.

## 4.2 Zero Forcing Sets

Let $G$ be a graph with vertex set $V(G)$ initially colored either blue or white. If $u$ is a blue vertex of $G$ and the neighborhood $N_G(u)$ of $u$ contains exactly one white vertex $v$, then we may change the color of $v$ to blue. This iterated procedure of coloring a graph is called zero forcing. A *zero forcing set* $B$ is a subset of vertices of $G$ such that if $G$ initially has all of the vertices of $B$ colored blue, then the zero forcing process can eventually color all of $V(G)$ blue. We let $z_k(G)$ denote the number of zero forcing sets of size $k$ of $G$.

It is easy to show that $z_1(G) > 0$ if and only if $G$ is a path graph. Partially motivated by this, the following conjecture was made by Boyer et. al. [2].

**Conjecture 4.4** ([2]). *If $G$ is an $n$-vertex graph, then for all $0 \leq k \leq n$, we have*

$$z_k(G) \leq z_k(P_n),$$

*where $P_n$ is the $n$-vertex path.*

Some small results towards this conjecture were given in [2] and [5], but overall almost nothing is known. **Update:** this result has also been proven whenever $G$ is an outerplaner graph by Menon and Singh [15].

In [5] we made a weaker conjecture.

**Conjecture 4.5** ([5]). *If $G$ is an $n$-vertex graph, then for all $0 \leq p \leq 1$, we have*

$$\sum_{k=1}^{n} z_k(G)p^k(1-p)^{n-k} \leq \sum_{k=1}^{n} z_k(P_n)p^k(1-p)^{n-k}.$$

Equivalently, this says that if we form a random set $B_p$ be including each vertex of $G$ independently with probability $p$, then the probability that $B_p$ is a zero forcing set of $G$ is at most that of it being a zero forcing set of $P_n$.

## 4.3 Card Guessing with Adversarial Shuffling

Consider the following game. We start with a deck of $mn$ cards consisting of $n$ different card types each appearing $m$ times (e.g. $m=4, n=13$ corresponds to a standard deck of cards). First, one of the players (Shuffler) shuffles the deck however they'd like. Then the other player (Guesser) sequentially guesses what the top card of the deck is. After each guess, the Guesser is told only whether their guess was correct or not, and then the top of the card is discarded. This game is called the *offline partial feedback model*, and the score at the end of the game is equal to the number of times Guesser correctly guesses a card type. One can also consider the *online partial feedback model* where Shuffler is allowed to reshuffle the remaining cards in the deck each time Guesser makes a guess.

**Question 4.6.** *Assuming $n \gg m$, can Guesser play in the offline partial feedback model so that they get $m + \omega(1)$ points in expectation? Can they play in the offline partial feedback model so that they get $m + \Omega(1)$ points in expectation?*

Simple strategies that Guesser can use in either model are to either guess a single card type each round, or to randomly guess a card type each round. Both strategies give Guesser $m$ points in expectation regardless of Shuffler's strategy. In [21] I came up with a strategy giving at least $m + 1/2$ points in the offline model (and an easy adaptation of the argument gives $m + e - 2$), as well as a strategy giving just a smidge more than $m$ in the online model; but the situation is pretty pitiful overall.

Note that in [6, 19], it is shown that if Shuffler shuffles the deck uniformly at random, then the Guesser can do is $m + \Theta(m^{1/2})$ points in expectation and that this is best possible. Thus this provide some reasonable benchmarks on how well one might be able to do here.

Finally, we note that one can consider variants of these problems for other "semi-restricted games" in the sense of [23].

# 5 Hall of Fame

## 5.1 ♢♢♢

- Wang and Zhao [25] for solving my original conjecture on ballot permutations; and Lin, Wang, and Zhao [13] for solving an even stronger version!

## 5.2 ♢

- Alon and Kravitz [1] for solving my problem with Greg Patchell about the number of CAT's one can pack into a cube filled with letters (and the extension to arbitrary words of distinct letters!).

- Pebody [20] for showing that every integer $n > 2$ has a bounded number of slowest tribonacci walks.

- Menon and Singh [15] for showing tight bounds for the number of zero forcing sets for trees.

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