Semi-restricted Rock, Paper, Scissors

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Joint work with Erlang Surya, Yuanfan Wang, Ji Zeng

Card Guessing

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Theorem (Diaconis-Graham, 1981)

For n fixed,

$$\mathcal{C}^{\pm}_{m,n} = m \pm c_n\sqrt{m} + o_n(\sqrt{m}).$$

Card Guessing

Theorem (Diaconis-Graham-X. He-S. 2020; J. He-Ottolini 2021)

For m fixed,

$$\mathcal{C}^{+}_{m,n} \sim H_m \log(n),$$
$$\mathcal{C}^{-}_{m,n} \sim \Gamma\left(1+\frac{1}{m}\right) n^{-1/m},$$

where H_m is the mth harmonic number and Γ is the gamma function.

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$$270,725$$

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Answer: a two player game played by Guesser and Shuffler.

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Let $C_{m,n}(\mathsf{G}, \mathsf{S})$ be the expected number of points Guesser scores when the two players follow strategies G, S.

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Theorem (S., 2021+)

If Shuffler wants to minimize the number of correct guesses and Guesser wants to maximize this, then under their optimal strategies G, S *we have*

$$C_{m,n}(\mathsf{G}, \mathsf{S}) = \log n + o_m(\log n).$$

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Theorem

There exists a strategy S *for Shuffler so that* $C_{m,n}(\mathsf{G}, \mathsf{S}) \leq \log n + o_m(\log n)$.

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Adversarial Card Guessing

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Interestingly, the greedy strategy is also the "unique" strategy which maximizes the number of correct guesses if Guesser tries to minimize their score.

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Consider a "semi-restricted" version of this game where they plan mn rounds of Matching Pennies and Bob must use each number exactly m times. This is exactly the same as the previous game!

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The unique optimal strategy for Rei is to play each option with probability 1/3 when every option remains, and to play the stronger card with probability 2/3 when two options remain. Moreover, Norman's advantage is $\Theta(\sqrt{n})$ if Rei plays each of Rock, Paper, and Scissors n times.

More General Games

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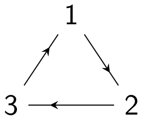
Given a digraph D, define the D-game by having two players simultaneously pick vertices of D each round.



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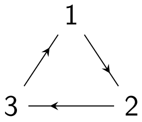

Given a non-negative integer vector $\vec{r}$, the *semi-restricted* D-game (with parameter $\vec{r}$) is defined by having players Rei and Norman iteratively play the D-game, with the restriction that Rei must play vertex v exactly $\vec{r}_v$ times.

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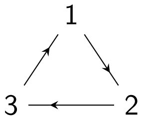

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Optimal Scores

Let $S_D(\vec{r})$ be the expected score for Norman in the semi-restricted D game is both players play optimally.

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$$S_D(\vec{r}) \geq \max_v \sum_{u \in N^+(v)} \vec{r}_u - \sum_{u \in N^-(v)} \vec{r}_u$$

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where $M = \sum_u \vec{r}_u$.

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Theorem (S.-Surya-Wang-Zeng; 2022+)

$$S_D(n, \dots, n) \geq \max_v (d^+(v) - d^-(v))n,$$

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Moreover, both bounds are best possible in general.

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Moreover, both bounds are best possible in general.

Corollary

If $d^+(v) = d^-(v)$ for all v (i.e. if D is Eulerian), then

$$0 \leq S_D(n,\ldots,n) \leq O_D(n^{1/2}).$$

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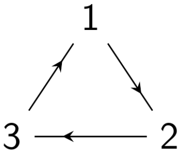
Given a digraph D, we define its skew adjacency matrix A by $A_{u,v} = +1$ if $u \to v$, $A_{u,v} = -1$ if $v \to u$, and $A_{u,v} = 0$ otherwise.



$$\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

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Theorem (S.-Surya-Wang-Zeng; 2022+)

If D is such that $Null(A) = span(\vec{1})$, then

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If D is an Eulerian tournament, then

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Question

Which digraphs satisfy $Null(A) = span(\vec{1})$?

Optimal Strategies

Theorem (S.-Surya-Wang-Zeng; 2022+)

If D is the directed path $1 \to 2 \to 3$, then a strategy for Rei is optimal if and only if she plays 3 with probability $1/2$ whenever she can.

$$\begin{array}{ccccc} \mathbf{1} & \longrightarrow & \mathbf{2} & \longrightarrow & \mathbf{3} \\ p & & 1/2 - p & & 1/2 \end{array}$$

Optimal Strategies

Question

Does every digraph D have an optimal strategy for Rei which is "oblivious", i.e. which only looks at which u Rei can play and ignores how many times she can play it?

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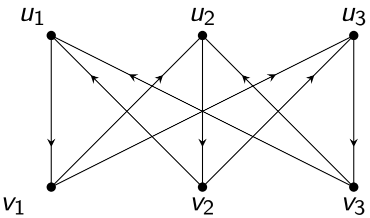
Theorem (S.-Surya-Wang-Zeng; 2022+)

The digraph depicted below does not have an oblivious optimal strategy for Rei.



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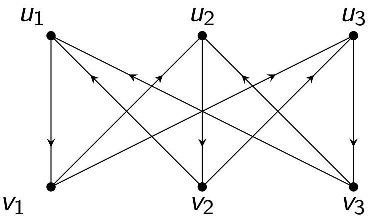

Theorem (S.-Surya-Wang-Zeng; 2022+)

There exist infinitely many Eulerian tournaments which do not have an oblivious optimal strategy for Rei.

Proofs: Bounds

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One can show that in expectation only $O_D(n^{1/2})$ turns remain after Rei runs out of some vertex to play. □

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$$S_D(\vec{r}) \leq \max_v \sum_{u \in N^+(v)} \vec{r}_u - \sum_{u \in N^-(v)} \vec{r}_u + O_D(M^{2/3}),$$

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$$\left(\max_v \sum_{u \in N^+(v)} \frac{\vec{r}_u}{\sum_w \vec{r}_w} - \sum_{u \in N^-(v)} \frac{\vec{r}_u}{\sum_w \vec{r}_w} \right) \cdot \sum_w \vec{r}_w.$$

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After something runs out, we expect the number of actions for any v to be at most $\vec{r}_v^{\,-1/2} \sum_u \vec{r}_u$

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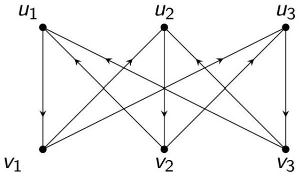
Either (1) Rei uses many p which are far from $\vec{1}$ (in which case $\|Ap\|_\infty$ is large) or (2) her strategy looks roughly uniform until something runs out.

Proofs: Strategies

Lemma

For RPS we have $S_D(\vec{r} - \delta_s) \leq S_D(\vec{r} - \delta_p) + 1$.

Proofs: Strategies



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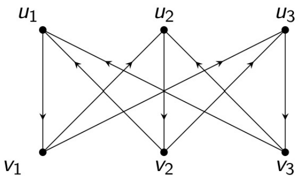

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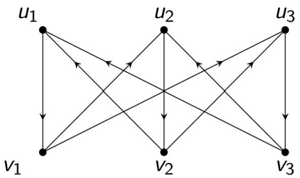

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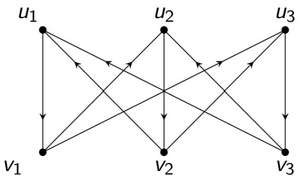

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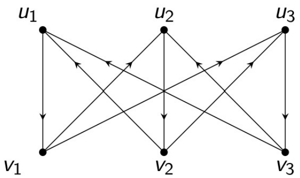
then there exist $\vec{r}$ with $S_D(\vec{r}) \gg \max_v \sum_{u \in N^+(v)} \vec{r}_u - \sum_{u \in N^+(v)} \vec{r}_u$.

Proofs: Strategies



Assume there was an oblivious optimal strategy for Rei with p_w the probability she picks w when every option is available. One can show for this D that if any w, w' has

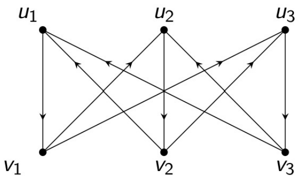
$$\sum_{u \in N^+(w)} p_u - \sum_{u \in N^-(w)} p_u < \sum_{u \in N^+(w')} p_u - \sum_{u \in N^-(w')} p_u,$$

then there exist $\vec{r}$ with $S_D(\vec{r}) \gg \max_v \sum_{u \in N^+(v)} \vec{r}_u - \sum_{u \in N^+(v)} \vec{r}_u$. One can show that such w, w' exist for all p, giving a contradiction.

Open Problems

Question

What are the optimal strategies for the semi-restricted D-game with D as below?



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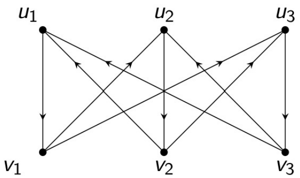

Question

What are the optimal strategies for directed paths?

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If D is an Eulerian digraph with at least one arc, do we have

$$S_D(n, \ldots, n) = \Theta_D(n^{1/2}).$$