

# Quotient Exercises

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1. Let  $\sim$  be the equivalence relation on  $[0, 1]$  with  $0 \sim 1$  and  $x \sim x$  for all  $x \in [0, 1]$ . Prove that  $[0, 1]/\sim$  is homeomorphic to the circle  $S^1$ .
2. Below we list several equivalence relations  $\sim$  for various spaces  $X$ . State (without proof) what familiar topological space  $X/\sim$  is homeomorphic to.
  - (a)  $X$  is a Euclidean disk  $D^2 = \{(x, y) \in \mathbb{R}^2 : |x|^2 + |y|^2 \leq 1\}$  with  $(x, y) \sim (x', y')$  iff  $|x|^2 + |y|^2 = |x'|^2 + |y'|^2$  (i.e. two points are identified if they have the same radius).
  - (b)  $X = \mathbb{R}^2$  with  $(x, y) \sim (x', y')$  iff  $|x|^2 + |y|^2 = |x'|^2 + |y'|^2$ .
  - (c)  $X = D^2$  with  $(x, y) \sim (x, y)$  for all  $(x, y) \in D^2$  and  $(x, y) \sim (x', y')$  whenever  $|x|^2 + |y|^2 = |x'|^2 + |y'|^2 = 1$  (i.e. all points on the boundary of  $D^2$  are identified).
  - (d)  $X = \mathbb{R}^n$  with  $x \sim y$  iff  $x - y \in \mathbb{Z}^n$  (Hint: we saw  $n = 1$  in class; solve  $n = 2$  and guess the pattern).